Flutter Control of a Two-dimensional Airfoil Using Wash-out Filter Technique

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Abstract: The wash-out filter (WF) technique is used to control the flutter of a two dimensional airfoil with cubic non-linearity in incompressible flow. Firstly, Hopf bifurcation theory is used to determine the point at which the nonlinear controller is introduced. The system is then transformed into Jordan canonical form, based on analysis of linearized eigenvalues of the system. Secondly, for the introduced WF controller, the linear control gain is determined according to Hopf bifurcation condition. The symbolic computing program of normal form direct method (NFDM) is also used to obtain the normal form of the controlled system. The non-linear control gain can be determined based on the relation of the type of bifurcation and the parameters of the normal form, to transform sub-critical Hopf bifurcation to be super-critical one. Lastly, numerical simulations are used to certify the validity of theoretical analysis, in which the amplitude of flutter or limit cycle of the controlled system is reduced greatly, comparing to the original system.

Key words: nonlinear airfoil; flutter control; wash-out filter (WF); Hopf bifurcation; normal form direct method (NFDM)

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摘 要: 研究应用 wash-out 滤波器技术对具有立方非线性俯仰刚度的二元机翼颤振的控制。首先,确定需要引入 Hopf 分岔的点,并在该点将原系统方程 Jordan 化;其次,对于引入的 wash-out 滤波控制器,先按 Hopf 分岔条件确定线性控制增益,再用规范型直接法得到受控系统的规范型,由分岔类型与规范型系数的关系确定非线性控制增益,从而将原系统的亚临界 Hopf 分岔变为超临界 Hopf 分岔;最后通过数值模拟验证了控制的有效性,并发现受控系统的颤振幅值(极限环大小)大大降低。

关键词: 非线性机翼; 颤振控制; wash-out 滤波器技术; Hopf 分岔; 规范型直接法
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For a flight airfoil, flutter is a dynamic aeroelastic instability that involves structure elastic, inertial and aerodynamic forces. Generally, flutter occurs when a critical flutter speed is exceeded. If flutter occurs in flight, the aircraft structure may fail. Therefore, it is important to predict the aeroelastic characteristics accurately to prevent the occurrence of flutter.

During the past decades, many aeroelastic analyses of flight vehicles have been performed. Typically, nonlinear aeroelastic responses include flutter, divergence, limit cycle oscillation (LCO) and chaotic motion. Zhao and Yang[1, 2] analyzed the LCO, period doubling motion and chaotic motion using the numerical integration and the harmonic balance method respectively. Liu[3] studied the typical bifurcation point using the successor function method. Price *et al.*[4] and Singh *et al.*[5], respectively, investigated the flutter characteristics in the time and the frequency domains by the describing function method. Ding *et al.*[6] improved the cell mapping method and applied it to global analysis of aeroelastic system with bilinear structural stiffness. Different types of motions including damped stable motion, LCO, complicated periodic motion, chaotic motion and divergent flut-

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ter were determined as a function of initial conditions (domains of attraction). Ding and Wang[7] studied the influence of structural coefficients on the topological structure of the Hopf bifurcation of the airfoil flutter. They found that depending on value of the linear stiffness coefficient, the Hopf bifurcation can be subcritical, a catastrophic type of flutter, and super-critical, a benign type of flutter. Obviously the sub-critical bifurcation should be avoided.

Bifurcation phenomenon is very complex. It can bring the dynamical system to different extents of danger and even disaster. Therefore, it is necessary to control the bifurcation phenomenon in real-time manner, to change the bifurcation from catastrophic type to benign one. Introducing intently the bifurcation controller can modify the dynamical characteristics of system, which includes postponing the occurrence of Hopf bifurcation, stabling the unstable bifurcation orbit, modifying the type or form of bifurcation orbit and controlling the chaotic motion through bifurcation control, and so on. The bifurcation controls have been widely used in such as biological medical engineering, aeronautic and aerospace engineering, and electricity system. The common used methods include the wash-out filter (WF) controller, linear and nonlinear feedback methods, frequency domain analysis, approaching approximation and normal form theory based method.

The WF method is expanded from linear or nonlinear feedback method. Utilizing the principle of bifurcation anti-control, the WF method can modify the system dynamics through introducing a new bifurcation. It has following advantages: simple controller structure; easy engineering implementation; small control cost in controlling bifurcation or chaotic; and being the same with multi-dimensional system and with certain robust. So the WF technique has been widely used in the engineering field. These include controlling the chaotic phenomenon of Lorenz system[8]; improving the complex dynamics of bifurcation and chaos of heart with unusually pulses, through adjusting the heart pulse and as well the alternative impulsion of the two heart chambers[9]; and controlling the chaos of the system under parametric disturbance[10].

In this paper, the active control on flutter of a two-dimensional airfoil system with structural cubic non-linearity is investigated by using the WF technique and the normal form directed method (NFDM). Firstly, the linear gain of the nonlinear controller is determined by using WF technique. Then the normal form of the controlled system is calculated by using the NFDM, to illustrate the relation of the topological bifurcation structure of the controlled system and the nonlinear gain of the nonlinear controller. With proper value of nonlinear gain, the sub-critical Hopf bifurcation can be suppressed by introducing a super-critical Hopf bifurcation intently, to improve the stability of flight airfoil.

1 Bifurcation of Airfoil with Cubic Non-linearity

For airfoil flying in incompressible flow, with viscous damp and cubic nonlinear pitch stiffness, the equations of motion of the two-degree-of-freedom aeroelastic system are described as[1]

$$\left.\begin{aligned} &\ddot{h} + 0.25\ddot{\alpha} + 0.1\dot{h} + 0.2h + 0.1Q\alpha = 0 \\ &0.25\ddot{h} + 0.5\ddot{\alpha} + 0.1\dot{\alpha} + k\alpha + e\alpha^3 - 0.04Q\alpha = 0 \end{aligned}\right\} \tag{1}$$

where h and α are the plunging displacement and the twist angle about the pitch axis. k, $e(=20)$ and Q are the linear pitch stiffness coefficient, nonlinear stiffness factor and the air speed, respectively. Flutter analysis[7, 11] showed that there exists a critical linear stiffness coefficient, $k_0 \approx 0.126$, that the case $k > k_0$ results a super-critical bifurcation and the case $k < k_0$ results a sub-critical bifurcation. Fig. 1 shows the global bifurcation diagram for sub-critical Hopf bifurcation ($k = 0.0816$). The linear bifurcation point, obtained from the stability analysis, is $B_1 = 1.5$. Nevertheless, the trivial solution of the system becomes local stable as the air speed $Q > Q_C \approx 0.9$, which means that a large enough disturbance or initial condition

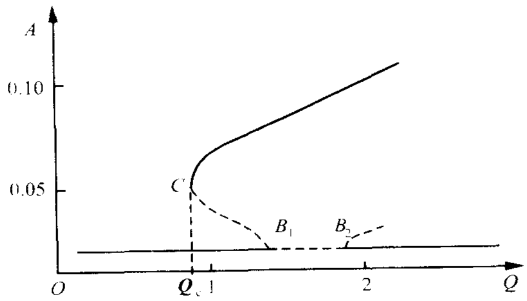


Fig. 1　Sub-critical Hopf bifurcation ($k = 0.0816$)

can induce that the trivial equilibrium loses its stability and a limit cycle with rather large amplitude will appear. Such a flutter is harmful to the safety of the flight airfoil. So the sub-critical Hopf bifurcation results a catastrophic type of flutter.

To suppress occurrence of the sub-critical Hopf bifurcation, the WF method will be adopted at $Q_C \approx 0.9$ to introduce a super-critical Hopf bifurcation. For the latter, the amplitude of limit cycle is increased gradually from zero. So the flutter is less harmful, or a benign type. Transform equation (1) into the state variable form as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + F(\boldsymbol{x}), \boldsymbol{x} = [\alpha \quad \dot{\alpha} \quad h \quad \dot{h}]^{\mathrm{T}} = [x_1 \quad x_2 \quad x_3 \quad x_4]^{\mathrm{T}} \tag{2}$$

where ($k = 0.0816$, $e = 20$)

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{4}{7}(0.26Q - 0.3264) & -0.228571 & 0.114286 & 0.057143 \\ 0 & 0 & 0 & 1 \\ \frac{4}{7}(0.0816 - 0.24Q) & 0.057143 & -0.228571 & -0.114286 \end{bmatrix}$$

$$F(\boldsymbol{x}) = 20 \begin{bmatrix} 0 \\ -16/7 \\ 0 \\ 4/7 \end{bmatrix} x_1^3$$

Obviously the trivial point ($\underline{\mathbf{0}}$) = [0　0　0　0] is equilibrium position of the system.

2　Design of the Nonlinear Controller Using the (WF) Technique

WF method is widely used in the field of controlling the multi-dimensional electric and aviation systems. By introducing the control input u to the system, a new super-critical Hopf bifurcation can be created[12]. In doing so, Eq. (2) becomes

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + F(\boldsymbol{x}) + \boldsymbol{B}u \tag{3}$$

where $\boldsymbol{B} = [0 \quad 0 \quad 0 \quad 1]^{\mathrm{T}}$. To keep the original equilibrium points unchanged, the control input u must satisfy the following constrains

$$y = \dot{\omega} = x_i - d\omega, u = g(y) = k_1 y + k_n y^3$$

where k_1 and k_n are the linear and non-linear control gains of the non-linear controller. x_i is the controlled state variable (which can be any component of the vector $\boldsymbol{x}$). d is the WF time constant and ω is a new introduced state variable. Substituting $Q = 0.9$ into the linearized matrix $\boldsymbol{A}$ of Eq. (3) yields eigenvalues

$$\lambda_{1,2} = -0.151839 \pm 0.316761\mathrm{i}$$
$$\lambda_{3,4} = -0.019590 \pm 0.410553\mathrm{i}$$

The trivial equilibrium point is still stable at present case because all the eigenvalues have negative real parts.

Applying transformation $\boldsymbol{x} = \boldsymbol{P}\boldsymbol{z}$, where $\boldsymbol{P}$ is the eigenvector corresponding to the eigenvalues of $\boldsymbol{A}$, the linear part of Eq. (3) can be transformed into real Jordan canonical form

$$\dot{\boldsymbol{z}} = \hat{\boldsymbol{A}}\boldsymbol{z} + \hat{F}(\boldsymbol{z}) + \hat{\boldsymbol{B}}u, \boldsymbol{z} = [z_1 \quad z_2 \quad z_3 \quad z_4]^{\mathrm{T}} \tag{4}$$

where

$$\boldsymbol{P} = \begin{bmatrix} 5.983747 & 2.328448 & 0.528905 & -0.457832 \\ -1.646128 & 1.541870 & 0.177603 & 0.226113 \\ -3.190044 & -2.106199 & -0.668367 & 0.862126 \\ 1.151535 & -0.690680 & -0.352639 & -0.443288 \end{bmatrix}$$

$$\hat{\boldsymbol{A}} = \boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} = \begin{bmatrix} -0.151839 & 0.316761 & 0 & 0 \\ -0.316761 & -0.151839 & 0 & 0 \\ 0 & 0 & -0.019590 & 0.410553 \\ 0 & 0 & -0.410553 & -0.019590 \end{bmatrix}$$

$$\hat{\boldsymbol{B}} = \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \\ \hat{B}_3 \\ \hat{B}_4 \end{bmatrix} = \boldsymbol{P}^{-1}\boldsymbol{B} = \begin{bmatrix} 0.238192 \\ 0.369420 \\ -2.976724 \\ 1.553095 \end{bmatrix}$$

$$\hat{F}(\boldsymbol{z}) = \boldsymbol{P}^{-1} F(\boldsymbol{P}\boldsymbol{z})$$

In Eq. (4), all coefficients of the linear items, B_i (i = 1, 2, 3 and 4), of the control input u is nonzero. So every state variable z_i is controllable[12]. Principally, the control procedure can be applied to all Jordan blocks. Considering the fact that the real parts of $\lambda_{3,4}$ is much closer to zero than that of $\lambda_{1,2}$, the control procedure will be applied to the Jordan block corresponding to $\lambda_{3,4}$ for smaller control cost.

By letting $\nu = z_4/B_4$ and $\xi = z_3 - B_3 z_4/B_4$, and evaluating z_1 and z_2 with values at the equilibrium point, *i. e.*, $z_1 = z_2 = 0$, the part of Eq. (4) corresponding to $\lambda_{3,4}$ can be transformed into

$$\left.\begin{aligned}\dot{\nu} &= f_1(\nu, \xi) + u\\ \dot{\xi} &= f_2(\nu, \xi)\end{aligned}\right\}\qquad(5)$$

where

$$\begin{cases} y = \dot{\omega} = \nu - d\omega \\ u = g(y) = k_l y + k_n y^3 \end{cases}$$

If the trivial solution of original system is stable, the time constant d is required only to be positive. In this paper d = 0.5 is taken. The linear gain k_l needs to satisfy the condition that the controlled system generates Hopf bifurcation, that is, the characteristic equation of the linear matrix of controlled equation has a pair of pure imaginary roots. Such a k_l can be calculated by Eq. (6)[12]. Whereas the nonlinear gain k_n, which assures the new introduced Hopf bifurcation to be super-critical, will be calculated by the normal form direct method (NFDM) later,

$$k_l = -(f_{11} + f_{22})[d^2 - d(f_{11} + f_{22}) + f_{11}f_{22} - f_{12}f_{21}]/[(d - f_{11} - f_{22})f_{21}]\qquad(6)$$

where

$$f_{11} = \left(\frac{\partial f_1}{\partial \nu}\right)\bigg|_{(\mathbf{0})} = 0.7672934$$

$$f_{12} = \left(\frac{\partial f_1}{\partial \xi}\right)\bigg|_{(\mathbf{0})} = -0.2643453$$

$$f_{21} = \left(\frac{\partial f_2}{\partial \nu}\right)\bigg|_{(\mathbf{0})} = 2.9799628$$

$$f_{22} = \left(\frac{\partial f_2}{\partial \xi}\right)\bigg|_{(\mathbf{0})} = -0.8064728$$

$$(\nu, \xi) = (0, 0) = (\mathbf{0})$$

Calculating Eq. (6) results k_l = 0.0106933. Transforming the controlled state variable to the original ones $\boldsymbol{x}$ and substituting the obtained formula into Eq. (3), one obtains

$$\left.\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{B}u\\ \dot{\omega} &= y = \nu - 0.5\omega = 1.8711686x_1 + \\ &\quad 1.1547622x_2 + 2.9139764x_3 - 0.5\omega\\ u &= g(y) = 0.0106933y + k_n y^3\end{aligned}\right\}\qquad(7)$$

$\boldsymbol{A}$, $\boldsymbol{F}$ and $\boldsymbol{B}$ in Eqs. (3) and (7) are identical.

Letting $\omega = x_5$ and taking it as a state variable, the Eq. (7) can be rewritten as

$$\dot{\boldsymbol{x}} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{H}(\boldsymbol{x}),\ \boldsymbol{x} = [x_1\quad x_2\quad x_3\quad x_4\quad x_5]^{\mathrm{T}}\qquad(8)$$

where $\boldsymbol{C}$ is a linear matrix and $\boldsymbol{H}(\boldsymbol{x})$ is the high-order term of $\boldsymbol{x}$. The coefficients of $\boldsymbol{H}(\boldsymbol{x})$ contain the nonlinear gain k_n.

3 Determination of the Nonlinear Gain by Normal Form Direct Method (NFDM)

According to the principle for designing controller, the linear matrix $\boldsymbol{C}$ of the controlled equation (8) should have a pair of eigenvalues with pure imaginary roots at Q = 0.9

$$\lambda_{1,2} = \pm 0.3958056\mathrm{i};\ \lambda_3 = -0.5391794,$$

$$\lambda_{4,5} = -0.1518389 + 0.3167613\mathrm{i}$$

Analysis indicates that the Hopf bifurcation conditions are also satisfied by this group of eigenvalues, whereas the nonlinear gain k_n of the controller needs to be determined further to let Hopf bifurcation to be super-critical. The analytical formulas of k_n and k_l for super-critical Hopf bifurcation were introduced in Ref. [12] for one-dimensional system. For two-dimensional system, k_n is determined only by numerical method due to the extensive complexity. To improve the application of WF technique in active control of the airfoil flutter, the NFDM will be applied to determine k_n at Q = 0.9.

Firstly, Q is also taken as a state variable. To keep the trivial solution unchanged, letting Q = 0.9 + x_6 yields the following equation

$$\left.\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{D}\boldsymbol{x} + \tilde{\boldsymbol{H}}(\boldsymbol{x})\\ \boldsymbol{x} &= [x_1\quad x_2\quad x_3\quad x_4\quad x_5\quad x_6]^{\mathrm{T}}\end{aligned}\right\}\qquad(9)$$

where $\boldsymbol{D}$ is a linear matrix. $\tilde{\boldsymbol{H}}(\boldsymbol{x})$, with coefficients containing k_n, is the high-order term of $\boldsymbol{x}$. The eigenvalues of the linear matrix $\boldsymbol{D}$ are calculated as

$$\lambda_1 = 0; \lambda_{2,3} = \pm 0.3958056i; \lambda_4 = -0.5391794$$
$$\lambda_{5,6} = -0.1518389 + 0.3167613i$$

Generally, the high-dimensional equation (9) can be reduced with the center manifold theorem, to obtain the lower-dimensional governing equations of dynamics on the center manifold. The normal form, which can be used to analyze the characteristic of the system's flutter behavior, can then be deduced by simplification procedure of the classic normal form method. During the deducing procedure, determining the reciprocal of Taylor series is demanded. So the computation is not only complex but also computing time and memory consuming when manipulating this procedure using a symbolic computer program, especially as the order of normal form increases. To overcome this shortage, the NFDM was used to study the flutter of a two-dimensional airfoil by the authors[7]. The deduction procedure of NFDM was performed by a symbolic compute program, which was developed by using a popular mathematic software, Maple. The explicit formulas for the normal form, up to an arbitrary order, and associated nonlinear transformation were conveniently presented in terms of the coefficients of the original differential equations.

The symbolic compute program developed in Ref.[7] is used in this paper to compute the normal form with parameters of the Eq. (9) in polar coordinate plane as

$$\left.\begin{aligned} \frac{dr}{dt} &= a\mu r + br^3 + h.o.t \\ \frac{d\varphi}{dt} &= \omega + cr^3 + e\mu + h.o.t \end{aligned}\right\} \tag{10}$$

where r and φ are the amplitude and phase of the bifurcation solution. The coefficients are $a = 0.0605779$, $b = 0.0215578 + 0.0095136k_n$, $\omega = -0.3958056$ (negative sign denotes the counter clockwise), $c = -0.0098422 + 0.0069838k_n$, $e = 0.0279768$ and $\mu = Q - 0.9$ (the bifurcation variable).

According to Hopf bifurcation theory, the type of bifurcation, *i.e.* the topological structure of the bifurcation solution, is determined by the sign of ab: $ab > 0$ yields a sub-critical bifurcation, whilst $ab < 0$ yields a super-critical bifurcation. To obtain a super-critical Hopf bifurcation at $\mu = 0$, the case $ab < 0$ is needed in designing controller. Because a is positive (and is independent of k_n, see Eq. (10)), a negative b (dependent on k_n, as shown in Fig.2) is required. One finds that the nonlinear gain of the WF controller should satisfy the condition $k_n < -2.2660045$.

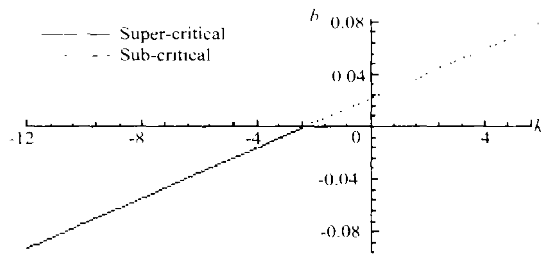


Fig.2 The relation between a and k_n

To verify the theoretical result, the steady-state bifurcation solution of the Eq. (10) can be obtained by setting $dr/dt = d\varphi/dt = 0$. The bifurcation curves are shown in Fig. 3 for various k_n (< -2.2660045). The curves suggest that the controlled airfoil with various k_n encounter super-critical Hopf bifurcation at $Q = 0.9$. The amplitude of the resulted flutter limit cycle r is also dependent on the nonlinear gain of WF, $|k_n|$. The larger the $|k_n|$, the smaller the r. This fact indicates that the amplitude of LC can be suppressed to a satisfactory level by increasing the control energy.

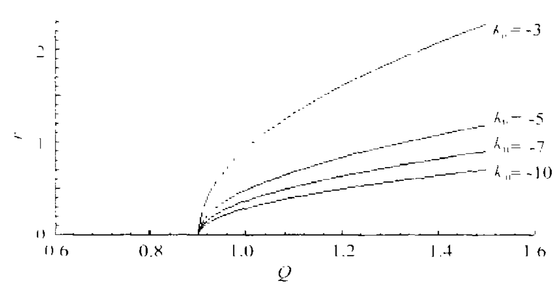


Fig. 3 Bifurcation curves of the controlled system for $d = 0.5$, $k_1 = 0.0106933$ and various k_n

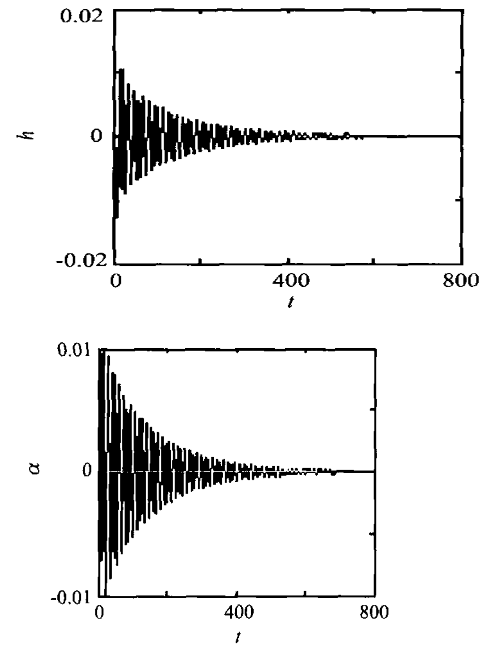
4 Numerical Simulation

Based on the above theoretical study, the change of the bifurcation point and type will be numerically simulated in the following, to verify the effect of the control technique on the airfoil system. The theoretical study shows that by introducing the nonlinear controller at air speed $Q = 0.9$, a super-critical Hopf bifurcation will be generated from the trivial equilibrium point. Take $k_n = -10$ for example, Eq. (7) becomes

$$\left.\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + F(\boldsymbol{x}) + \boldsymbol{B}(0.0106933y - 10y^3) \\ \dot{\omega} &= y = 1.8711686x_1 + 1.1547622x_2 + \\ &\quad 2.9139764x_3 - 0.5\omega \end{aligned}\right\} \qquad (11)$$

where $\boldsymbol{B}$ and $\boldsymbol{A}$ are the same as these in Eqs. (2) and (3). From the bifurcation diagrams in Fig. 3, it is known that the equilibrium point $(\underline{\mathbf{0}}, Q)$: (a) is asymptotically stable focus for $Q < 0.9$; (b) becomes a center singularity for $Q = 0.9$; and (c) is unstable focus for $Q > 0.9$, and a asymptotically stable limit cycle is bifurcated.

For the case $Q = 0.8 < 0.9$, *i. e.*, before the bifurcation point, the phase trajectories and time histories of the controlled system, with initial condition $(x_1, x_2, x_3, x_4, \omega)_0 = (\alpha, \dot{\alpha}, h, \dot{h}, \omega)_0 = (0.02, 0, 0, 0, 0)$, are presented in Fig. 4. Obviously the disturbed motion will converge at the trivial equilibrium point after a certain period of time.



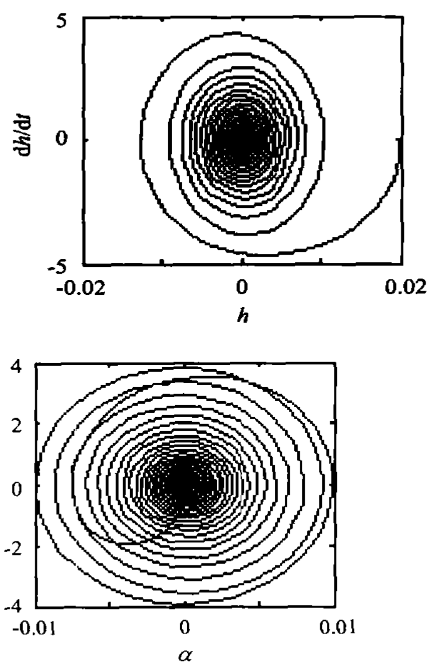
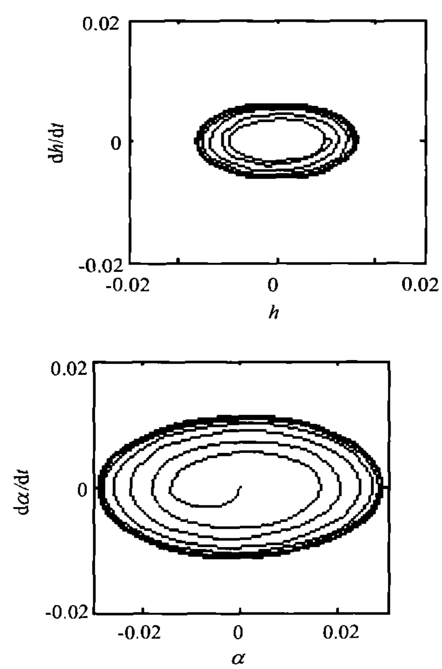


Fig. 4 Phase trajectories and time histories of the controlled system before bifurcation
($Q = 0.8, (x_1, x_2, x_3, x_4, \omega)_0 = (0.02, 0, 0, 0, 0)$)

For the case $Q = 1.6 > 0.9$, *i. e.*, after the bifurcation point, a asymptotically stable limit cycle is bifurcated from the trivial equilibrium position, with gradually increasing amplitude. The calculated results for initial condition (0.01, 0, 0, 0, 0) are shown in Fig. 5.

To study the suppressing effect by using the controller on the flutter response, the responses in



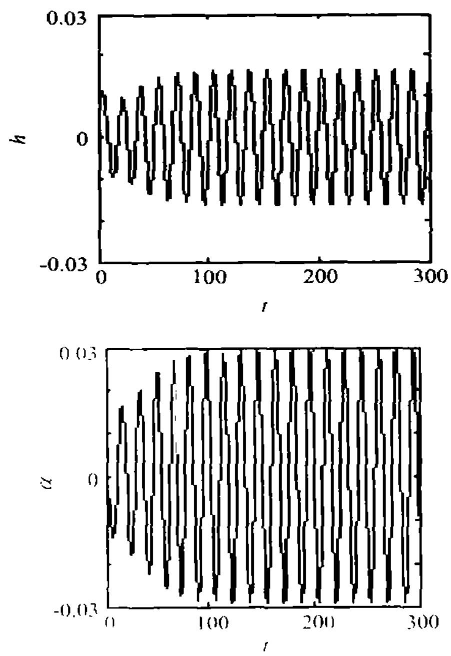


Fig. 5 Phase trajectories and time histories of the controlled system after bifurcation
($Q = 1.6, (x_1, x_2, x_3, x_4, \omega)_0 = (0.01, 0, 0, 0, 0)$)

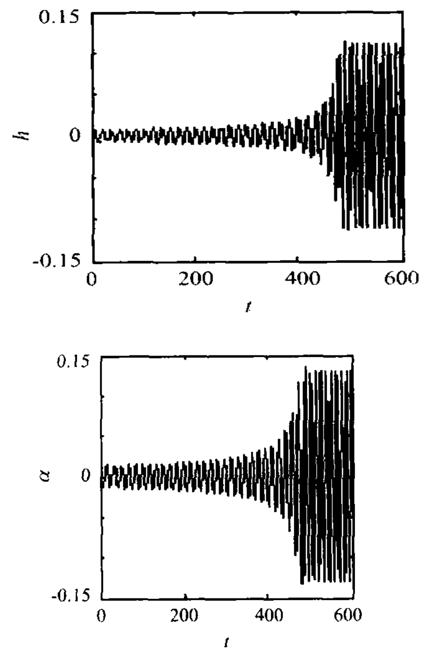
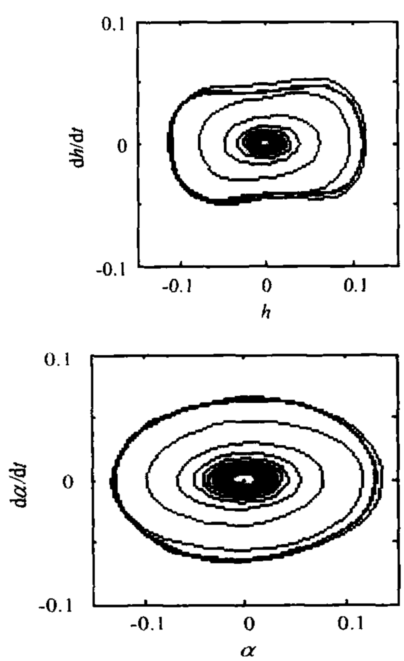


Fig. 6 Phase trajectories and time histories of the original system after bifurcation

h and α directions in Fig. 5 are compared with that of the original system at $Q = 1.6$ (note that the sub-critical Hopf bifurcation occurs at $Q = 1.5$). The latter is shown in Fig. 6 for initial condition (0.01, 0, 0, 0). One finds that the amplitude of the flutter response is reduced eight to ten times after the controller is applied. So introduction of the controller based on WF technique can greatly



improve the stability of the flight airfoil system.

5 Conclusions

Applying nonlinear controller based on the WF technique, the type of the airfoil flutter can be changed topologically, for most case, from sub-critical Hopf bifurcation, a catastrophic type, to super-critical Hopf bifurcation, a benign type. Generally the nonlinear controller is introduced ahead but near the original bifurcation point. By doing so, besides that the bifurcation type is changed, the amplitude of the flutter response or limit cycle can also be reduced greatly. So the stability of the flight airfoil system is improved.

In designing the nonlinear controller, the symbolic computing program of the norm formal direct method is used to calculate the normal form of the controlled system conveniently, and to present the explicit relation between the nonlinear gain and topological flutter type of the controlled system. The validity of the nonlinear controller is verified by numerical simulation.

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